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# MODELING AN ELECTRIC MOTOR IN 1-D

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BUTLER ANALYSES

## INTRODUCTION

Quite often the dynamicist will be faced with having an electric drive motor as a link in the elastic path of a structure such that the motor's characteristics must be taken into account to properly represent the dynamics of the primary structure. He does not want to model it so accurately that he could get detailed stress and displacements in the motor proper, but just sufficiently to represent its inertia loading and elastic behavior from its mounting bolts to its drive coupling. This paper describes how the rotor and stator of such a motor can be adequately modeled as a colinear pair of beams.

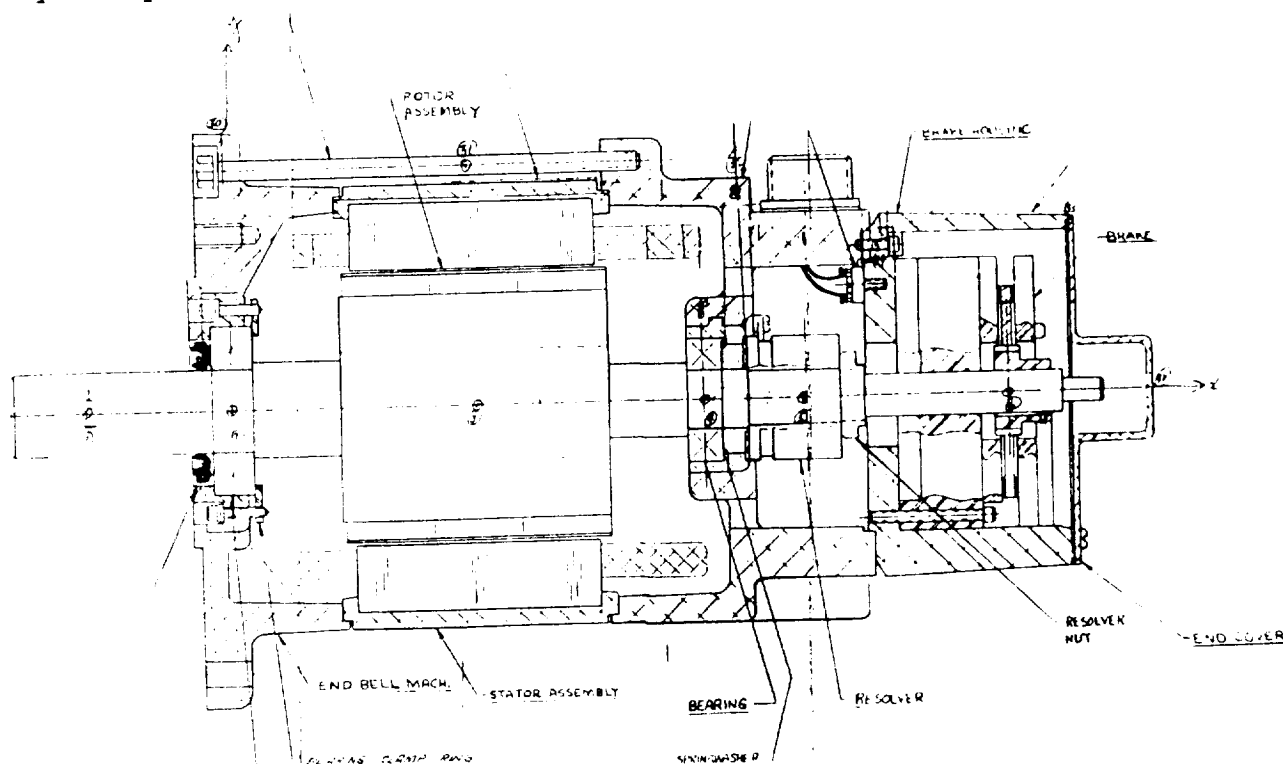


Figure 1

## MODELING AN ELECTRIC MOTOR IN 1 - D

### PLAN

Figure 1 shows an assembly drawing of a motor equipped with a disk brake. The application in which this motor was incorporated required that the brake be set; consequently, the electric coupling from the armature to the stator field was not modeled. The discussion of the modeling will be taken up in four parts: the rotor, the stator, the brake and the mount.

The overall scheme is to locate grid points at the bearings, at the concentrations of mass, at the brake, and at the mating interfaces. Six (6) grid points were assigned to the rotor and 8 grid points were assigned to the stator. See Table 1. Between the grid points the structure undergoes several changes of section. Each section is considered separately then the stiffness for an equivalent prismatic bar is computed for the sequence of sections between grid points.

One of the intriguing features about finite element analysis is that parts can occupy the same physical location yet can still remain disjoint. That feature will be employed here. The stator is concentric with the rotor so the centroid of the stator is coincident with the centroid of the rotor. Thus their individual beam models will be colinear.

### EQUIVALENCE

An example of one equivalent bar will be developed for the rotor and another for the stator. The rotor between the bell housing and the armature center of gravity has 3 separate sections: the bearing journal, intermediate shaft and armature. The three sections are arrayed in line one after the other. This is an elastic series situation so equivalence is found by summing compliances ( $Z$ ). The formulas for equivalent area and equivalent bending inertia are:

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$$\frac{E A_{\text{equiv}}}{L_{\text{total}}} = \left( \sum z_{\text{area}} \right)^{-1} = \left[ \frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} + \frac{L_3}{A_3 E_3} \right]^{-1}.$$

Since the copper windings are surrounded by a preponderance of steel it is permissible to assume that the material in the armature can be represented as steel thus all E's are the same and the equivalent area becomes:

$$\begin{aligned} A_{\text{equiv}} &= L_{\text{total}} \left[ \left( \frac{L}{A} \right)_1 + \left( \frac{L}{A} \right)_2 + \left( \frac{L}{A} \right)_3 \right]^{-1} \\ &= 4.889 \left[ \frac{1}{\pi/4} \left( \frac{.389}{1.3785^2} + \frac{1.67}{2.25^2} + \frac{2.83}{3.831^2} \right) \right]^{-1} = 5.28 \end{aligned}$$

Similarly, the computation of the equivalent bending inertias proceeds with

$$\begin{aligned} \frac{E I_{\text{equiv}}}{L_{\text{total}}} &= \left( \sum z_I \right)^{-1} = \left[ \frac{L_1}{E_1 I_1} + \frac{L_2}{E_2 I_2} + \frac{L_3}{E_3 I_3} \right]^{-1}. \\ I_{\text{equiv}} &= L_{\text{total}} \left[ \left( \frac{L}{I} \right)_1 + \left( \frac{L}{I} \right)_2 + \left( \frac{L}{I} \right)_3 \right]^{-1} = \\ &= 4.889 \left[ \frac{1}{\pi/64} \left( \frac{.389}{1.3785^4} + \frac{1.670}{2.25^4} + \frac{2.83}{3.831^4} \right) \right]^{-1} = 1.29 \end{aligned}$$

$$J_{\text{equiv}} = 2 I_{\text{equiv}}.$$

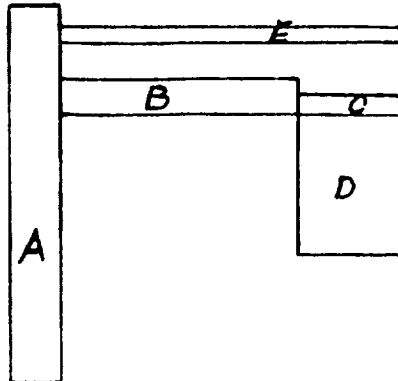


Figure 2

# MODELING AN ELECTRIC MOTOR IN 1 - D

The approach to equivalent properties for the stator is similar to that for the rotor except that the sections are not solid, the array is partially in parallel and partially in series, and it involves distinct materials, as seen in Figure 2. Note that sections C & D act in parallel; therefore the net combined stiffness "K" is  $K_{CD} = K_C + K_D$ , and the net compliance is  $Z_{CD} = 1/K_{CD}$ . Section B acts in series with the CD combination, therefore the net combined compliance is  $Z_{BCD} = Z_B + Z_{CD}$ . Section E acts in parallel with the inner combination of BCD, therefore the net elasticity is obtained by adding stiffnesses.  $K_{BCDE} = K_E + K_{BCD} = K_E + Z_{BCD}^{-1}$  and  $Z_{BCDE} = 1/K_{BCDE}$ . Finally, section A acts in series with the combined sections BCDE; i.e.  $Z_{net} = Z_A + Z_{BCDE}$ . The equivalent sectional properties can be obtained from  $Z_{net}$ . The numerical statistics for the components of this stator bar example are given in Table 3. Use of these data in the combining formulas is carried out just for stator equivalent area as follows.

$$\begin{aligned} K_{CD}^{area} &= \frac{A E}{L} \Big|_C + \frac{A E}{L} \Big|_D = \frac{\pi/4(7.12^2 - 6.75^2) \times 10^7}{2.689} + \\ &\quad \frac{\pi/4(6.75^2 - 4.58^2) 3 \times 10^7}{2.689} = \\ &= \frac{\pi \times 10^7}{4 \times 2.689} (5.132 + 3 \times 24.586) = 2.304 \times 10^8. \\ Z_{CD}^{area} &= 4.340 \times 10^{-9}. \end{aligned}$$

Combining item B in series with the CD combination gives

$$\begin{aligned} Z_{BCD}^{area} &= Z_B^{area} + Z_{CD}^{area} = \frac{L_B}{A E_{al}} + Z_{CD}^{area} = \\ &= \frac{2.007}{\pi/4(7.38^2 - 6.5^2) \times 10^7} + 4.34 \times 10^{-9} = 2.526 \times 10^{-8} \\ K_{BCD}^{area} &= (Z_{BCD}^{area})^{-1} = 3.95 \times 10^7. \end{aligned}$$

Item E combines in parallel with item BCD.

$$K_{BCDE}^{area} = \frac{4(\pi/4)(5/16)^2 \times 3 \times 10^7}{4.53} + 3.958 \times 10^7 = 4.162 \times 10^7.$$

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$$Z_{BCDE}^{area} = (K_{BCDE}^{area})^{-1} = 2.403 \times 10^{-8}.$$

And now section A can be combined in series with the current result.

$$\begin{aligned} Z_{net}^{area} &= \frac{L_A}{A A_{al} E_{al}} + Z_{BCDE}^{area} = \frac{0.62}{(7.5^2 - \frac{\pi}{4} 2/85^2) \times 10^7} + 2.403 \times 10^{-8} \\ &= 2.567 \times 10^{-8} = \frac{L_{total}}{E_{al} A_{equiv}}. \\ A_{equiv} &= \frac{E_{al}}{L_{total} Z_{net}^{area}} = 19.347. \end{aligned}$$

Similar calculations yield equivalent area moment of inertia  $I_{equiv}$  as was shown for the rotor bar example.

### COLLATERAL

These operations were performed for the rest of the rotor and stator. This completes the complement of properties for equivalent prismatic bars. It will be mentioned, only in passing, that the mass can be well modeled automatically (except for torsional inertias) by calling for COUPMASS. The assignment of torsional mass moment of inertia per grid point must be manually determined and entered as CMASS2 elements. The next topic concerns the mating of the rotor to the stator at the bearings. This motor was designed with ball bearings which can absorb thrust. Therefore, the two bearings link the rotor and stator without connecting any rotations. This was represented as Multi-Point-Constraints (MPC's) in all 3 translations, between the two rotor bearing points, numbers 2 & 4 and the corresponding stator points, numbers 30 & 33.

The topic that needed particular study was the disc brake. The following discussion will refer to Figure 3.

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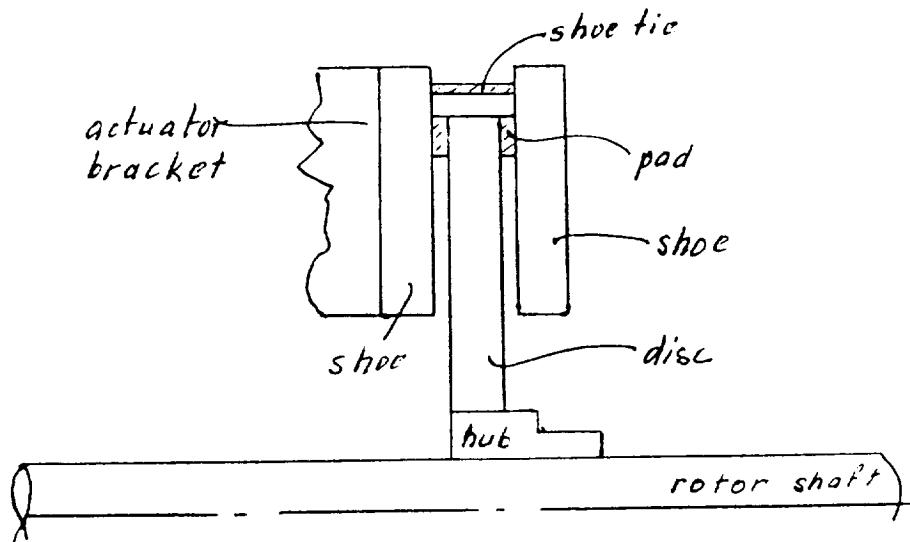


Figure 3

Two shoes bind on the brake disc to hold the rotor shaft. When the brake is actuated, the load path from shaft to stator is all that is of concern. How the brake force is exerted is extraneous to the finite element modeling of the braked condition (if stresses are of no concern). The as-braked load path is from the shaft at the aft end of the brake disc into the hub, into the disc and out of the forward shoe to the brake bracket via the pad then into the resolver bell. Thus there is no contribution to the stiffness of this load path from the after shoe. The modeling starts by incorporating the hub into the bar of the rotor shaft. The stator bar begins with the disc and connects in series with the forward pad. Then the shoe is a series component in the equivalent bar extending from the resolver bell through the actuator bracket and the shoe. It was difficult to get material properties for the asbestos pad so it was assumed that the asbestos could be reasonably represented by limestone.

Finally, consideration must be given to the connection of this one dimensional model to the outside world. The drive shaft will connect through a keyway into a power shaft of a reduction gear or similar; so this poses no difficulty in linking 1 - D to

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1 - D. With the stator bell it is a different matter. The motor casing attaches at either end bell or through feet along its length to a 2 - D or 3 - D parent. In this model it is through the forward bell. If MPC's or rigid elements are chosen to make this connection at the forward bell one must review the MPC used to tie the ball bearings together. The ball bearing MPC must retain the stator degrees of freedom as independent, so that they will be available to be picked up for further tying to the mounting bolts. If instead of constraints, bar connections are used, then consideration must be exercised so as not to short circuit the forward bell elasticity with an overly stiff representation. Nor should the bars be so limber as to introduce wobble into the connection.

### SUMMARY

The statistics for this 1 - D model of an electric motor are:

Grid Points - 6 in the rotor, 8 in the stator = 14.

Bars - 5 in rotor, 8 in stator = 13

MPC - 3

Torsional Mass Elements CMASS2 - 14

An exploded plot of these 2 colinear elements in Figure 4 illustrates the connections at bearings, brake, and externals.

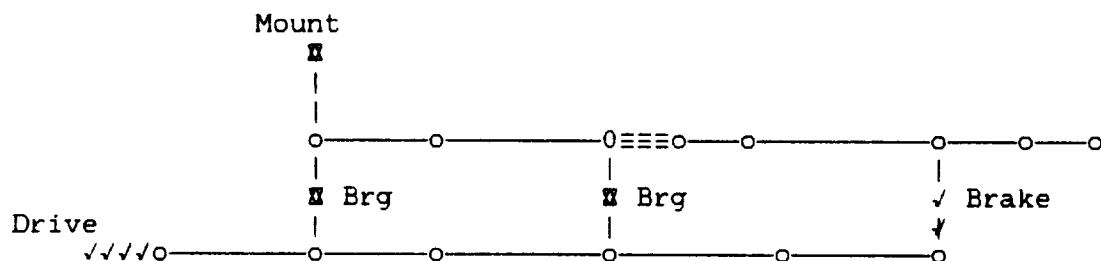


Figure 4

## MODELING AN ELECTRIC MOTOR IN 1 - D

### CONCLUSION

It has been shown that the necessary characteristics of an electric motor can be incorporated into a dynamic model by means of a lean bar model without having to resort to a full blown three dimensional model.



MODELING AN ELECTRIC MOTOR IN 1 - D  
TABLE OF GRID POINTS

ROTOR

GP	LOCATION
1	Drive Shaft Coupling Keyway
2	Center Line of Bell Housing with Bearings
3	Center of Gravity of Rotor Armature
4	Bearing at Brake End
5	Center of Gravity of Resolver
6	Center Line of Brake Disc

STATOR

30	Center line of Bell Housing with Bearings
31	Center of Gravity of Stator Coil
32	Center Line of Outer Resolver Bell
33	Bearing at Brake End
34	Resolver End Bell
35	Brake Housing
36	End Cover
37	Brake Shoe Plate

Table 1

MODELING AN ELECTRIC MOTOR IN 1 - D  
TABLE OF COMPONENT STATISTICS

ROTOR COMPONENT STATISTICS

DESCRIPTION	DIAMETER	LENGTH	MATERIAL
1/2 Bearing Journal	1.3875	0.389	Steel
Intermediate Shaft	2.25	1.670	Steel
1/2 Armature	3.831	2.830	Steel/Copper
Total		4.889	Steel

Table 2

STATOR BAR COMPONENT STATISTICS

ITEM	DESCRIPTION	DIAMETER OD/ID	LENGTH	MATERIAL
A	Bell	7.5 / 2.85 Square Round	0.62	Aluminum
B	Shell	7.38/2.85	2.007	Aluminum
C	Stator Shell	7.12/6.75	2.689	Aluminum
D	Stator	6.75/4.58	2.689	Steel/Copper
E	4 Bolts	$\frac{5}{16}$	4.53	Steel
	Total		4.889	Aluminum

Table 3